Geometry – Module 1, Topic 1 – Conditional and Biconditional Statements

Hi, guys. Welcome to Geometry. This topic's going to focus on conditional and biconditional statements. Your knowledge of logic and reasoning is going to come in handy for you during this lesson. You ready to get started? Let's go.

Okay, now, before we dive into the meat of the lesson, the conditional and biconditional statements, I want to warm you up a little bit with this Venn diagram. Presented with this Venn diagram, where are a few different ways you could interpret it? Well, you could say, "All people that live in Virginia live in the US," and that'd be a true statement based on how my Venn diagram is situated.

Now, I could also say, "If you live in Virginia, then you live in the US," and that'd also be true based on how my Venn diagram is situated. Now, that second stated that I used, that "if, then" statement, that's a conditional statement. That's what we're going to really get into here.

Okay, so let's take that conditional statement apart a bit. If you live in Virginia, then you live in the US. The part of that statement, this phrase that immediately follows "if," that's known as my hypothesis. Let me get my pen out here. We use the symbol p to represent our hypothesis. The part of the sentence that follows "then," "you live in the US," we call that our conclusion. We use the variable q to represent our conclusion.

Now, if we wanted to use symbols, like you know we do a lot of times with math, to represent a conditional statement, I would use the symbols p, an arrow, and then q. That arrow represents "implies," or that you simply have an if, then statement. Symbolically, a conditional statement is, "If p, then q." In this case, because p represents you live in Virginia and q represents you live in the US, then for this example, this symbol would mean, "If you live in Virginia, then you live in the US." That's a conditional statement.

Now, there are a lot of modifications, and by that, I just mean changes, that you can make to a conditional statement. For example, let's move this out of our way here so that we can see that statement. Okay. We're going to continue to work with our conditional statement, "If you live in Virginia, then you live in the US." The converse, which you can probably tell here from the symbols, that I've reversed q and p, I've reversed my hypothesis and conclusion. That's what the converse does to a conditional statement.

If I wanted to write the converse of that statement, it would read, "If you live in the US, then you live in Virginia." Now, let's think about that for a little bit. Is that actually a true statement? It's not. The truth value of that statement is false because let's think about it, if you live in the US, then you live in Virginia. Well, the US has 50 states, so if you live in the US, you could live in Texas, Florida, Maryland. There are a lot of other states that you could live in. All those examples that I'm giving you of why this statement is false, we call those counter-examples, which just means that those are statements, or those are examples that prove that that modification is false. The takeaway from this that I want you to get is that the converse reverses the hypothesis and conclusion, and you can see symbolically it's q, arrow, p, and that sometimes when you make changes to your conditional statement, and even sometimes your conditional statement itself, is not always true. Sometimes it's false. You can come up with counter-examples to prove that that statement is false.

Okay, all right, let's make another change here. Let's make another modification. Here, we've got an inverse that we're going to do. We see that if you live in Virginia, then you live in the US. Notice these symbols here. In layman's terms, you could call them "squiggles." They're technical known as "tildes," but I have a tilde-p or a squiggle-p, an arrow, and a squiggle-q. When you see that symbol, it means that you're negating something. Or, in other words, you're going to put "not" in that sentence. Or, if "not" is already there, then you would take it out. For this example, the inverse of our conditional statement is, "If you do not live in Virginia, then you do not live in the US." We negated the hypothesis, and we negated the conclusion. All right, now think about that for a second. Is that a true statement? That's not, you're right. The truth value of that is also false.

Again, what are some counter-examples that you could give me to prove that that statement is false? Well, the inverse says, "If you do not live in Virginia, you do not live in the US," then tell me some other states that you could live in and still live in the US. Well, you could live in New York, New Jersey, New Mexico, Idaho, there are tons of other states that you could live in that are not Virginia but still in the US. The takeaway from this one, the inverse negates the hypothesis, and it negates the conclusion, and again, when you write the inverse of a statement, sometimes it's false, and you can give counter-examples to prove that the inverse is false.

All right, there's one more modification that we're going to talk about, and that's the contrapositive. Notice symbolically it's represented by the symbols squiggle, because it's just more fun to say than "tilde," but ~q, arrow, ~p. Notice here that you've reversed your hypothesis and conclusion, and you've also negated it. There's two operations you performed to your conditional statement to write your contrapositive. Let's take a look at the contrapositive. Our conditional is, "If you live in Virginia, then you live in the US." The contrapositive of that is, "If you do not live in the US, then you do not live in Virginia." You notice how we've reversed our hypothesis and our conclusion, and we've also negated it. That's how you write the contrapositive.

Now, let's think about the truth value of this one. Is that statement true? It is a true statement. Let's break it down a bit. If you do not live in the United States, then you do not live in Virginia. Well, that has to be true because if you did live in Virginia, that would mean that you did live in the US. The takeaway from this one, the contrapositive reverses and negates the hypothesis and the conclusion of our conditional statement, and you saw here our contrapositive was actually true. Now, if for some reason, like with the others, if our contrapositive is false, you have to give counter-examples to prove that that statement is false.

For these next few examples, I'm going to show you a conditional statement, and then I'm going to show you a modification. I want you to identify the modification, so tell me is it the converse, the inverse, or the contrapositive, and then I want you to determine its truth value. Is it true, or is it false? Now, if it's false, keep in mind that you should have some counter-examples to be able to prove that it's false. Now, let's take these one at a time.

All right, so here's our conditional statement. I'll give you a second to read that. "If today is Thursday, then tomorrow is Friday." I'm going to show you the modification. I want you to take a minute and see if you can tell me is it the converse, the inverse, or the contrapositive. All right, let's see how you did on that part. "If today is not Thursday, then tomorrow is not Friday." All right, I'm going to switch to my pen here. Today is Thursday, that's p. Tomorrow is Friday, that's q. Today is not Thursday, well that's the negation of p, so that's ~p or squiggle-p. Then tomorrow is not Friday. Okay, well, tomorrow is not

Friday, that's the negation of q. That means this modification, ~p, arrow, ~q, and that's the inverse, if you remember those symbols. The modification we have here is the inverse of our conditional statement. Now, is that true? If today is not Thursday, then tomorrow is not Friday. Let's take a look at that. That is a true statement. If today isn't Thursday, then tomorrow isn't Friday because Friday only follows Thursday.

Okay, now let's take a look at this conditional statement. I'll give you a minute to read it. Okay. If angle C measures 100 °, then angle C is not acute. All right, now I'm going to reveal the modification, and I want you to determine if it's the converse, the inverse, or the contrapositive. All right, let's see how you did here. My modification says, "If angle C is not acute, then angle C measures 100 °." I kind of tried to trick you a little on this one. I hope you got through it okay. Okay, so let's break down the conditional first. Angle C measures 100 °. That's p, that's my hypothesis. Angle C is not acute, that's q. Notice that q, our hypothesis, already has "not." I don't want you to think that any time you see "not," that automatically the modification that's coming has to be the inverse or has to be the contrapositive because those are the modifications that negate the conditional statement. It's not always so. Notice on this one. Our modification starts out with, "If angle C is not acute." That's actually just q again. Even though it has "not," it's just q.

Then angle C measures 100 °. That's p. Our modification is q arrow p. The reverse of the hypothesis of the conclusion, so it's the converse. Let's check out the truth value here. Is this actually a true statement. "If angle C is not acute, then angle C measures 100 °." Let's reveal this. Let's get that red dot out of the way. Switch to our arrow. That is false. "If angle C is not acute, then angle C measures 100 °" isn't a true statement. There are a lot of other measures that angle C could have and still not be an acute angle. It could be 90 ° and be a right angle. It could measure 150 ° and still not be an acute angle. That, like we saw here, that converse is not a true statement. We just gave some examples of some counter-examples to prove that.

Okay, let's keep going here. It is now your turn to write the converse, the inverse, and the contrapositive of a conditional statement. I want you to really take some time and practice actually writing it all out. I've given you a conditional statement here. Again, determine its truth value. All right, let's see how you did here.

If angle B measures 120 °, then angle B is obtuse. Writing the converse of that statement, you should have written, "If angle B is obtuse, then angle B measures 120 °," because, again, to write the converse, you're simply reversing the hypothesis and the conclusion. Our hypothetical is p, conclusion is q, so to write the converse, you just reverse that. All right. Now, let's check out the truth value of this one. That was false. Again, like we just practiced dealing with those angle measures in the previous example, here on this one, the converse of angle B is converse, then angle B measures 120 ° isn't necessarily so. It could measure anything above 90 to below 180 °. It doesn't necessarily have to be 120. It could be 119 °, 121 °. Okay, all right. Let's check this next one out.

We've got to write the inverse. Okay, so again, we still had our same conditional statement. Let's see how you did with writing the inverse. If angle B does not measure 120 °, then angle B is not obtuse. That should have been how you wrote the inverse because again, switch to our pen here, p is our hypothesis, q is the conclusion, and you know the inverse negates the hypothesis and the conclusion. We do have that here. Angle B does not measure 120 °, that angle B is not obtuse. Now, what did you get for that truth value? Let's see here. False. Let's think about what some of your counter-examples might have been. If angle B does not measure 120, then angle B is not obtuse. You could have given me any other angle measure besides 120 ° that's still classified as an obtuse angle. Again, 150 °, 160 °, anything besides 120.

All right, got one more to look at here. You had to write the contrapositive. Let's see how you did. Let's reveal this. If angle B is not obtuse, then angle B does not measure 120 °. Again, just to talk about how you got that, p is your hypothesis, q is your conclusion, and remember the contrapositive reverses and negates the hypothesis and the conclusion. Here we have angle B is not obtuse, which is the negation of q. Then angle B does not measure 120 °, which is the negation of p. Then for that truth value, let's see if you got the same thing I did. Let's reveal that. That is a true statement. Let's think about that. If angle B is not obtuse, it can't measure 120 ° because if it did, then it would be an obtuse angle.

All right, now before we leave this topic, we need to talk a little bit about a biconditional statement. Now, what a biconditional is, it's actually a definition. It's a fact. How you know that you can write a biconditional statement, there's 2 requirements. The conditional and the converse of your statement both have to be true. If they're both true, then you can write a biconditional statement. The symbolic form for a biconditional is p with the double-headed arrow and then q. Now, the double-headed arrow has a different meaning than just our regular old arrow. It means "if and only if." Sometimes you'll see this abbreviated as "if and only if." When you're reading a biconditional statement, it'll state your hypothesis, if and only if, and then your conclusion. The conditional and the converse both must be true, and a biconditional is just a definition.

Take a look at this. "If today is Monday, then yesterday was Sunday." Whoop. Okay, so that's our conditional statement. Is that true? It is true. If today is Monday, then yesterday was Sunday. So far, make sure we got our pen here, it's checking out. Now let's look at the converse of that statement. If yesterday was Sunday, then today is Monday. That's also true. We had our hypothesis, get the pen back here, and our conclusion, and to write the converse, we had to reverse the hypothesis and the conclusion. Again, we did end up with a true statement, "If yesterday was Sunday, then today is Monday." That is true. Because both the converse and the conditional statement were both true, you can write a biconditional statement. This is what that looks like. Let's get that out of our way. "Today is Monday if and only if yesterday was Sunday."

Let's get our pen back here. I know we keep switching back and forth. Today is month. There's p. If and only if. There's our double-headed arrow. Yesterday was Sunday. There's q. That's an example of a biconditional statement. The conditional and the converse, both true, and then you can join your hypothesis and conclusion with "if and only if."

Okay, let's take a look this. I do believe it is time for you to try one. I'm going to give you a statement, and I want you to tell me can it be written as a biconditional. If it can, go ahead and write that biconditional statement. If it can't, I want you to figure out why. Okay, so here we go. I'll give you a minute to read that. Can this be written as a biconditional statement? If it can, write it as a biconditional. If it can't, explain why. All right, let's see how you did on this.

The statement itself, the conditional statement, "If angle D measures 90 °, then angle D is a right angle." That's a true statement. Our conditional statement, it's true. We need to see if the converse is also true. Let's take a look at that. If angle D is a right angle, then angle D measures 90 °. We reversed our hypothesis and conclusion. Did we end up with a true statement? We did because if angle D is a right angle, it must measure 90 °. That's also true. Because our conditional and our converse were both true, then you can write your biconditional statement. Let's take a look.

Angle D measures 90 ° if and only if angle D is a right angle. You see we've joined our hypothesis, here's p, and our conclusion, here's q, with "if and only if." Remember, we said that's that double-headed arrow. All right. Great job working with conditional and biconditional statements and learning how to apply your logic and reasoning skills to determine if those statements were true or to determine if they were false, and learning how to generate those counter-examples to prove when a statement is actually false. Bye.